#198

UCLA Packet Radio Temporary Note #5 (Analytic)
PRT 136

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ON GIANT STEPPING

IN

PACKET RADIO NETWORKS

Recently Network Analysis Corporation has suggested, as a result of their simulation experiments, that it is advantageous for a given repeater to hop over as many intermediate repeaters as possible in its attempt to route packets to the station in a centralized network. (This we call giant stepping). Of course, giant stepping is accomplished by increasing the transmitting power of repeaters.

In this note, we suggest a simple model in which we prove that unlimited giant stepping is clearly not optimal in uniform distributed networks and that in fact a critical propagation radius r_c can easily be found which is the optimum step size to be used. (The issue regarding optimum step size in the centralized networks created by the single station philosophy still lacks an analytic treatment).

The Model

We take the simplest possible model (not unlike the one used by Abramson in Packet Radio Note No. 49) in which we assume a uniform density of interfering traffic. Specifically, we assume that there is a packet generating source at a rate of λ packets per second per unit area; we will assume that this uniform density applies to the entire real plane and that this is the traffic carried by repeaters. Traffic is originated by and destined for repeaters; in this sense, all repeaters are also

stations, i.e., we have point-to-point traffic. Secondly, we assume that all destinations are equally likely (a truly distributed net). Thirdly, we assume that every point in the real plane is a potential repeater. Each packet transmission relayed by one repeater will be heard by all other repeaters within a radius r. Among all those repeaters that hear this first repeater, only one (as determined by the routing procedure) will be allowed to relay this given transmission. We assume that the labeling is carried out such that this relay repeater lies exactly at a distance r from the first repeater and in the direction toward the destination. Fourth, we assume that the average delay required to successfully transmit the packet a distance r (i.e., one hop) is given by the function $T(\lambda(r))$ where $\lambda(r)$ represents the total interfering traffic within the radius r of the receiving repeater.

We wish to study the performance of a "tagged" packet which is required to travel a distance D through the network from its source to its destination. For convenience let us assume that D is an integral multiple of r. As a consequence, the tagged packet must travel a total of D/r hops as it passes through this network and we will assume that at each hop the average delay is given by $T(\lambda(r))$. This of course assumes that each repeater transmits out to a radius r and similarly that each repeater is disturbed by all sources with the radius r. See Figure 1.

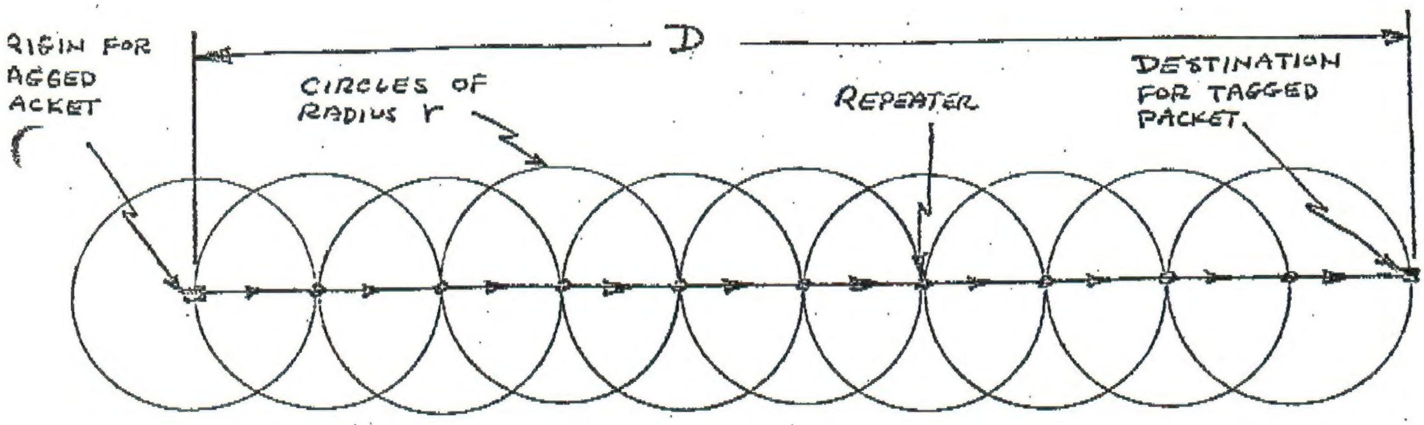

Figure 1
The sequence of repeaters relaying the tagged packet. Here $D = 10r$

Of course, most of the interesting analytical work is involved in finding the function $T(\lambda(r))$. This for example has been discussed in the work by Kleinrock and Tobagi [1].

The Optimization Problem

Our task, simply, is to find that critical value of r namely r_c which will minimize the total delay that a packet experiences in its travels through the net. Clearly if r approaches zero, then, the number of hops will grow to infinity and this will be a poor solution. Similarly, if r is very large, then the interfering traffic which the repeater is subject to will eventually drive it into saturation and the throughput will drop to zero thereby increasing T to infinity. These two intuitive limits suggest that there does exist a finite value of r (as opposed to the unlimited value of r suggested by NAC for single station nets). In this section we find that critical value (r_c).

Specifically, we wish to solve the following optimization problem

$$\min_{r} \frac{D}{r} T(\lambda(r)) \qquad (1)$$

where of $\frac{D}{r} T(\lambda(r))$ is simply the expected delay our packet experiences. The following simple condition therefore determines the value of r which we are seeking

$$\boxed{\frac{dT(\lambda(r))}{dr} = \frac{T(\lambda(r))}{r}} \tag{2}$$

The solution to this last equation will give us r_c. (We ignore the trivial, yet annoying requirement that D be an integral multiple of r ; this is acceptable if $D >> r_c$) . In Figure 2 we show the graphical interpretation of this result.

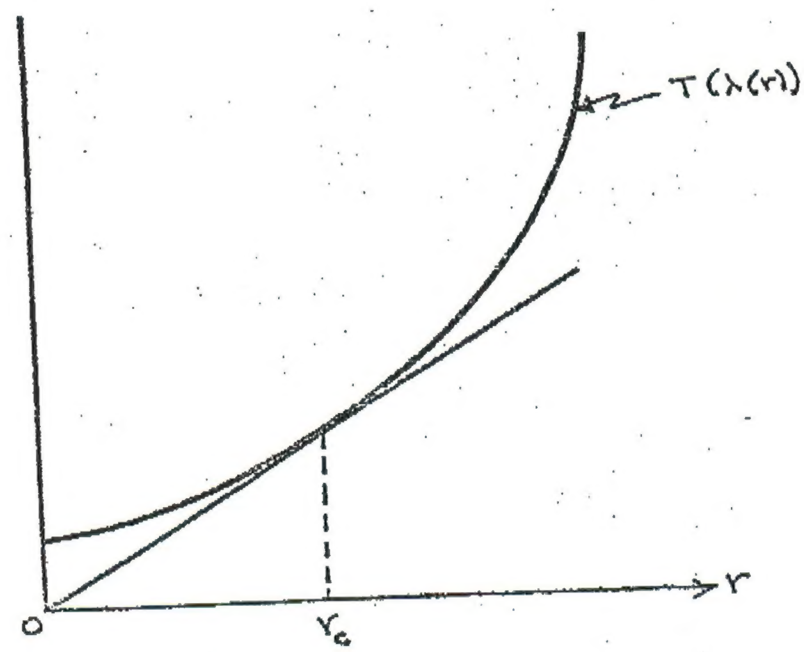

Figure 2
The Optimum Radius, r_c : a Graphical Interpretation

Here we have shown $T(\lambda(r))$ as a convex function although this is certainly not necessary as, for example, shown in Figure 3 below.

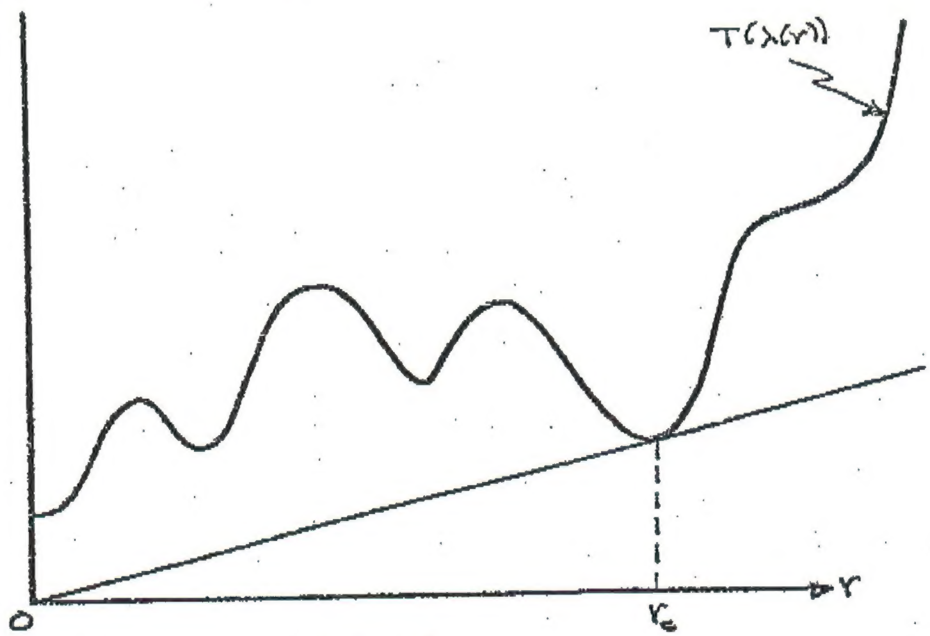

Figure 3

The Optimum Radius, r_c, for an Arbitrary $T(\lambda(r))$

In this figure (3), we show a clearly unrealistic shape for $T(\lambda(r))$. The solution for r_c is simply obtained as the first intersection of a ray from the origin beginning with slope zero and increasing in slope until the ray hits the T curve. We also note that the average total delay is simply given by

$$\text{Average Total Delay} = D \left. \frac{dT(\lambda(r))}{dr} \right|_{r = r_c} \qquad (3)$$

Thus, we see that our solution is good for arbitrary $T(\lambda(r))$.

A Simple Example

As a simple example, let us assume that the per-hop delay is simply given by the M/M/1 queueing formula [2], that is,

$$T(\lambda(r)) = \frac{1}{\mu C - \lambda(r)} \tag{4}$$

where the average packet length is taken as $1/\mu$ bits, the channel capacity is C bits per second and, of course, the average packet generating rate in the radius r is simply $\lambda(r)$. Due to our assumption of a uniform packet generation rate, we have simply

$$\lambda(r) = \lambda\pi r^2 \tag{5}$$

Using these expressions in Equation 2, we find immediately that

$$r_c = \sqrt{\frac{\mu C}{3\pi\lambda}} \tag{6}$$

and that the total average delay (minimized) is

$$\begin{aligned} \text{Minimum Average Total Delay} &= \frac{DT(\lambda(r_c))}{r_c} \\ &= \frac{3D}{2\mu C}\sqrt{\frac{3\pi\lambda}{\mu C}} \\ &= \frac{D}{2\pi\lambda}(r_c)^3 \end{aligned} \tag{7}$$

The behavior of the total average delay is shown in Figure 4, which is simply the graph of

$$\frac{DT(\lambda(r))}{r} = \frac{D}{r(\mu C - \lambda\pi r^2)} \tag{8}$$

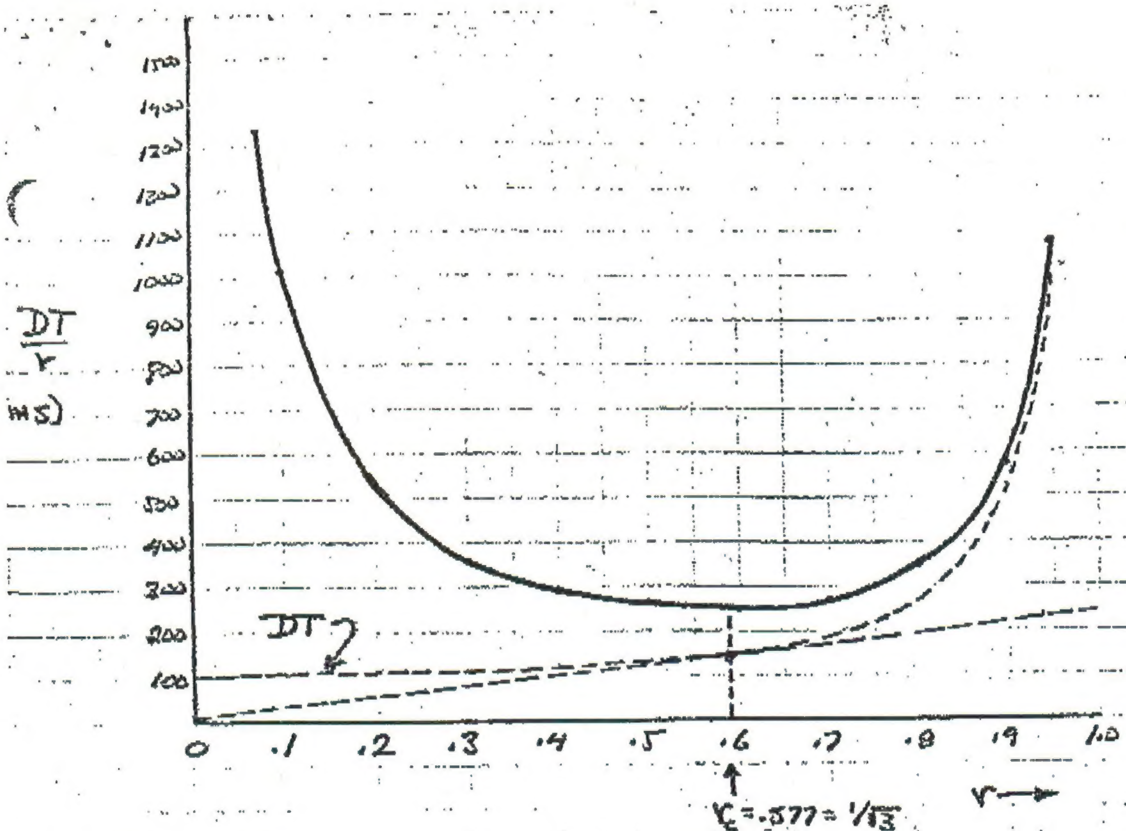

Figure 4

Average Total Delay for $D = 10$, $1/\mu C = 10$ ms, $\mu C/\lambda\pi = 1$

For this sketch, we have chosen $D = 10$, $1/\mu C = 10$ms (the packet transmission time) and $\mu C/\lambda\pi = 1$. We note how flat the minimum is for DT/r. At the critical radius $r_c = 1/\sqrt{3} \cong 0.577$, we find the minimum delay is $DT/r_c \cong 260$ms. We also show (dashed) the sketch of DT and the ray which first touches it (at $r = r_c$, of course).

Conclusions

In this (albeit simple) model of a point-to-point packet radio net, we have been able to show that unlimited giant stepping is undesirable but that there is a critical step size (we are fighting the temptation to generate a mythological name for this critical radius) as determined by Equation 2.

References

[1] Kleinrock, L. and F. Tobagi, "Random Access Techniques for Data Transmission Over Packet Switched Radio Networks", paper presented at the National Computer Conference, 1975.

[2] Kleinrock, L., "Queueing Systems, Vol. 1: Theory", Wiley-Interscience, 1975.